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# Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays

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(LHCb Collaboration)

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Observations of exotic structures in the  $J/\psi p$  channel, which we refer to as charmonium-pentaquark states, in  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays are presented. The data sample corresponds to an integrated luminosity of  $3 \text{ fb}^{-1}$  acquired with the LHCb detector from 7 and 8 TeV  $pp$  collisions. An amplitude analysis of the three-body final state reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the  $J/\psi p$  mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of  $4380 \pm 8 \pm 29 \text{ MeV}$  and a width of  $205 \pm 18 \pm 86 \text{ MeV}$ , while the second is narrower, with a mass of  $4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$  and a width of  $39 \pm 5 \pm 19 \text{ MeV}$ . The preferred  $J^P$  assignments are of opposite parity, with one state having spin  $3/2$  and the other  $5/2$ .

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**Introduction and summary.**—The prospect of hadrons with more than the minimal quark content ( $q\bar{q}$  or  $qqq$ ) was proposed by Gell-Mann in 1964 [1] and Zweig [2], followed by a quantitative model for two quarks plus two antiquarks developed by Jaffe in 1976 [3]. The idea was expanded upon [4] to include baryons composed of four quarks plus one antiquark; the name pentaquark was coined by Lipkin [5]. Past claimed observations of pentaquark states have been shown to be spurious [6], although there is at least one viable tetraquark candidate, the  $Z(4430)^+$  observed in  $\bar{B}^0 \rightarrow \psi' K^- \pi^+$  decays [7–9], implying that the existence of pentaquark baryon states would not be surprising. States that decay into charmonium may have particularly distinctive signatures [10].

Large yields of  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays are available at LHCb and have been used for the precise measurement of the  $\Lambda_b^0$  lifetime [11]. (In this Letter, mention of a particular mode implies use of its charge conjugate as well.) This decay can proceed by the diagram shown in Fig. 1(a), and is expected to be dominated by  $\Lambda^* \rightarrow K^- p$  resonances, as are evident in our data shown in Fig. 2(a). It could also have exotic contributions, as indicated by the diagram in Fig. 1(b), which could result in resonant structures in the  $J/\psi p$  mass spectrum shown in Fig. 2(b).

In practice, resonances decaying strongly into  $J/\psi p$  must have a minimal quark content of  $c\bar{c}uud$ , and thus are charmonium pentaquarks; we label such states  $P_c^+$ , irrespective of the internal binding mechanism. In order to

ascertain if the structures seen in Fig. 2(b) are resonant in nature and not due to reflections generated by the  $\Lambda^*$  states, it is necessary to perform a full amplitude analysis, allowing for interference effects between both decay sequences.

The fit uses five decay angles and the  $K^- p$  invariant mass  $m_{Kp}$  as independent variables. First, we tried to fit the data with an amplitude model that contains 14  $\Lambda^*$  states listed by the Particle Data Group [12]. As this did not give a satisfactory description of the data, we added one  $P_c^+$  state, and when that was not sufficient we included a second state. The two  $P_c^+$  states are found to have masses of  $4380 \pm 8 \pm 29 \text{ MeV}$  and  $4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$ , with corresponding widths of  $205 \pm 18 \pm 86 \text{ MeV}$  and  $39 \pm 5 \pm 19 \text{ MeV}$ . (Natural units are used throughout this Letter. Whenever two uncertainties are quoted, the first is statistical and the second systematic.) The fractions of the total sample due to the lower mass and higher mass states are  $(8.4 \pm 0.7 \pm 4.2)\%$  and  $(4.1 \pm 0.5 \pm 1.1)\%$ , respectively. The best fit solution has spin-parity  $J^P$  values of  $(3/2^-, 5/2^+)$ . Acceptable solutions are also found for additional cases with opposite parity, either  $(3/2^+, 5/2^-)$  or  $(5/2^+, 3/2^-)$ . The best fit projections are shown in Fig. 3. Both  $m_{Kp}$  and the peaking structure in  $m_{J/\psi p}$  are reproduced by the fit. The significances of the lower mass and

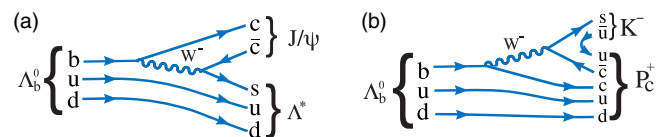


FIG. 1 (color online). Feynman diagrams for (a)  $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$  and (b)  $\Lambda_b^0 \rightarrow P_c^+ K^-$  decay.

\*Full author list given at end of the article.

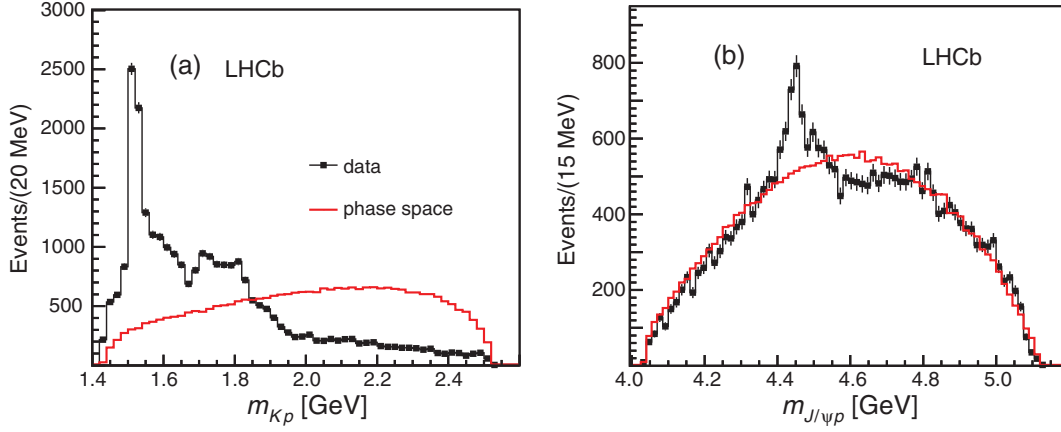


FIG. 2 (color online). Invariant mass of (a)  $K^- p$  and (b)  $J/\psi p$  combinations from  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.

higher mass states are 9 and 12 standard deviations, respectively.

*Analysis and results.*—We use data corresponding to  $1 \text{ fb}^{-1}$  of integrated luminosity acquired by the LHCb experiment in  $pp$  collisions at 7 TeV center-of-mass energy, and  $2 \text{ fb}^{-1}$  at 8 TeV. The LHCb detector [13] is a single-arm forward spectrometer covering the pseudorapidity range,  $2 < \eta < 5$ . The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the  $pp$  interaction region [14], a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes [15] placed downstream of the magnet. Different types of charged hadrons are distinguished using information from two ring-imaging Cherenkov detectors [16]. Muons are identified by a system composed of alternating layers of iron and multiwire proportional chambers [17].

Events are triggered by a  $J/\psi \rightarrow \mu^+ \mu^-$  decay, requiring two identified muons with opposite charge, each with transverse momentum,  $p_T$ , greater than 500 MeV. The dimuon system is required to form a vertex with a fit  $\chi^2 < 16$ , to be significantly displaced from the nearest  $pp$  interaction vertex, and to have an invariant mass within 120 MeV of the  $J/\psi$  mass [12]. After applying these requirements, there is a large  $J/\psi$  signal over a small background [18]. Only candidates with dimuon invariant mass between  $-48$  and  $+43$  MeV relative to the observed  $J/\psi$  mass peak are selected, the asymmetry accounting for final-state electromagnetic radiation.

Analysis preselection requirements are imposed prior to using a gradient boosted decision tree, BDTG [19], that separates the  $\Lambda_b^0$  signal from backgrounds. Each track is required to be of good quality and multiple reconstructions of the same track are removed. Requirements on the individual particles include  $p_T > 550$  MeV for muons,

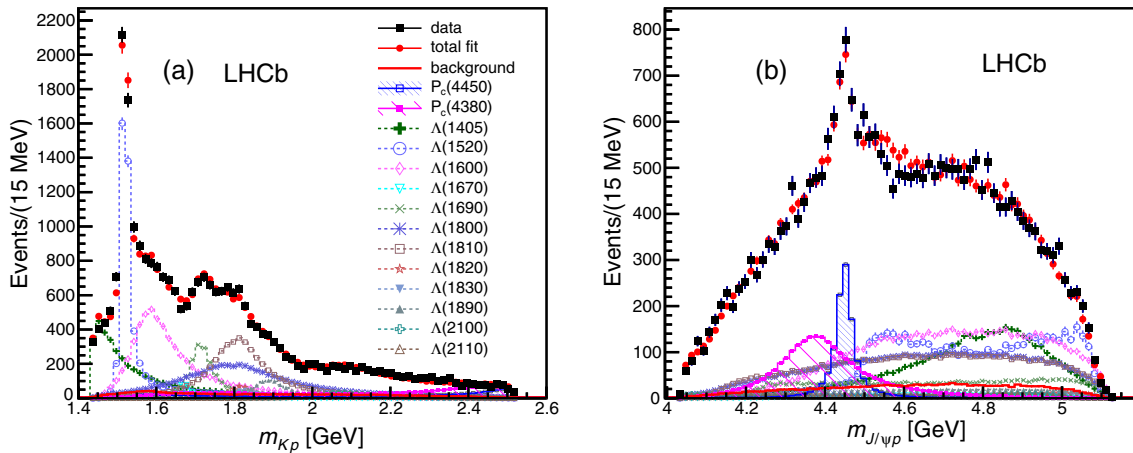


FIG. 3 (color online). Fit projections for (a)  $m_{Kp}$  and (b)  $m_{J/\psi p}$  for the reduced  $\Lambda^*$  model with two  $P_c^+$  states (see Table I). The data are shown as solid (black) squares, while the solid (red) points show the results of the fit. The solid (red) histogram shows the background distribution. The (blue) open squares with the shaded histogram represent the  $P_c(4450)^+$  state, and the shaded histogram topped with (purple) filled squares represents the  $P_c(4380)^+$  state. Each  $\Lambda^*$  component is also shown. The error bars on the points showing the fit results are due to simulation statistics.

and  $p_T > 250$  MeV for hadrons. Each hadron must have an impact parameter  $\chi^2$  with respect to the primary  $pp$  interaction vertex larger than 9, and must be positively identified in the particle identification system. The  $K^-p$  system must form a vertex with  $\chi^2 < 16$ , as must the two muons from the  $J/\psi$  decay. Requirements on the  $\Lambda_b^0$  candidate include a vertex  $\chi^2 < 50$  for 5 degrees of freedom, and a flight distance of greater than 1.5 mm. The vector from the primary vertex to the  $\Lambda_b^0$  vertex must align with the  $\Lambda_b^0$  momentum so that the cosine of the angle between them is larger than 0.999. Candidate  $\mu^+\mu^-$  combinations are constrained to the  $J/\psi$  mass for subsequent use in event selection.

The BDTG technique involves a “training” procedure using sideband data background and simulated signal samples. (The variables used are listed in the Supplemental Material [20].) We use  $2 \times 10^6$   $\Lambda_b^0 \rightarrow J/\psi K^- p$  events with  $J/\psi \rightarrow \mu^+\mu^-$  that are generated uniformly in phase space in the LHCb acceptance, using PYTHIA [21] with a special LHCb parameter tune [22], and the LHCb detector simulation based on GEANT4 [23], described in Ref. [24]. The product of the reconstruction and trigger efficiencies within the LHCb geometric acceptance is about 10%. In addition, specific backgrounds from  $\bar{B}_s^0$  and  $\bar{B}^0$  decays are vetoed. This is accomplished by removing combinations that when interpreted as  $J/\psi K^+ K^-$  fall within  $\pm 30$  MeV of the  $\bar{B}_s^0$  mass or when interpreted as  $J/\psi K^- \pi^+$  fall within  $\pm 30$  MeV of the  $\bar{B}^0$  mass. This requirement effectively eliminates background from these sources and causes only smooth changes in the detection efficiencies across the  $\Lambda_b^0$  decay phase space. Backgrounds from  $\Xi_b$  decays cannot contribute significantly to our sample. We choose a relatively tight cut on the BDTG output variable that leaves  $26\,007 \pm 166$  signal candidates containing 5.4% background within  $\pm 15$  MeV ( $\pm 2\sigma$ ) of the  $J/\psi K^- p$  mass peak, as determined by the unbinned extended likelihood fit shown in Fig. 4. The combinatorial background is modeled with an exponential function and the  $\Lambda_b^0$  signal shape is parametrized by a double-sided Hypatia function [25], where the signal radiative tail parameters are fixed to values obtained from simulation. For subsequent analysis we constrain the  $J/\psi K^- p$  four-vectors to give the  $\Lambda_b^0$  invariant mass and the  $\Lambda_b^0$  momentum vector to be aligned with the measured direction from the primary to the  $\Lambda_b^0$  vertices [26].

In Fig. 5 we show the “Dalitz” plot [27] using the  $K^-p$  and  $J/\psi p$  invariant masses-squared as independent variables. A distinct vertical band is observed in the  $K^-p$  invariant mass distribution near  $2.3 \text{ GeV}^2$  corresponding to the  $\Lambda(1520)$  resonance. There is also a distinct horizontal band near  $19.5 \text{ GeV}^2$ . As we see structures in both  $K^-p$  and  $J/\psi p$  mass distributions we perform a full amplitude analysis, using the available angular variables in addition to the mass distributions, in order to determine the

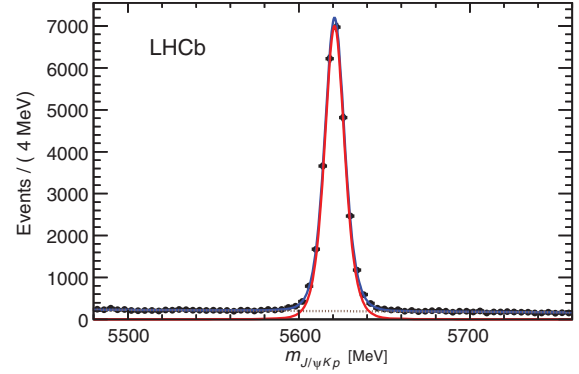


FIG. 4 (color online). Invariant mass spectrum of  $J/\psi K^- p$  combinations, with the total fit, signal, and background components shown as solid (blue), solid (red), and dashed lines, respectively.

resonances present. No structure is seen in the  $J/\psi K^-$  invariant mass.

We consider the two interfering processes shown in Fig. 1, which produce two distinct decay sequences:  $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ ,  $\Lambda^* \rightarrow K^- p$  and  $\Lambda_b^0 \rightarrow P_c^+ K^-$ ,  $P_c^+ \rightarrow J/\psi p$ , with  $J/\psi \rightarrow \mu^+\mu^-$  in both cases. We use the helicity formalism [28] in which each sequential decay  $A \rightarrow BC$  contributes to the amplitude a term

$$\begin{aligned} \mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} D_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\phi_B, \theta_A, 0)^* R_A(m_{BC}) \\ = \mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} e^{i\lambda_A \phi_B} d_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\theta_A) R_A(m_{BC}), \end{aligned} \quad (1)$$

where  $\lambda$  is the quantum number related to the projection of the spin of the particle onto its momentum vector (helicity) and  $\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC}$  are complex helicity-coupling amplitudes describing the decay dynamics. Here,  $\theta_A$  and  $\phi_B$  are the polar and azimuthal angles of  $B$  in the rest frame of  $A$  ( $\theta_A$  is known as the “helicity angle” of  $A$ ). The three arguments of Wigner’s  $D$  matrix are Euler angles describing the rotation of the initial coordinate system with the  $z$  axis along the

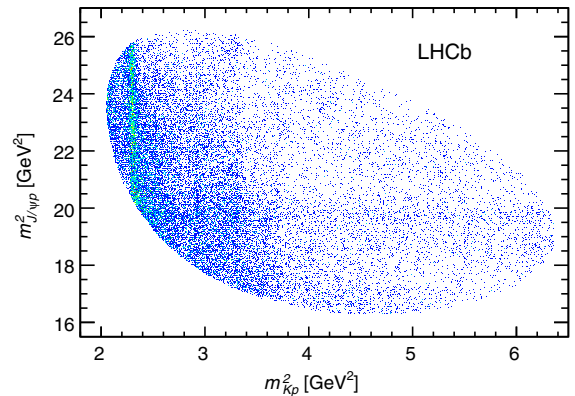


FIG. 5 (color online). Invariant mass squared of  $K^- p$  versus  $J/\psi p$  for candidates within  $\pm 15$  MeV of the  $\Lambda_b^0$  mass.

helicity axis of  $A$  to the coordinate system with the  $z$  axis along the helicity axis of  $B$  [12]. We choose the convention in which the third Euler angle is zero. In Eq. (1),  $d_{\lambda_A, \lambda_B - \lambda_C}^{J_A}(\theta_A)$  is the Wigner small- $d$  matrix. If  $A$  has a non-negligible natural width, the invariant mass distribution of the  $B$  and  $C$  daughters is described by the complex function  $R_A(m_{BC})$  discussed below; otherwise  $R_A(m_{BC}) = 1$ .

Using Clebsch-Gordan coefficients, we express the helicity couplings in terms of  $LS$  couplings ( $B_{L,S}$ ), where  $L$  is the orbital angular momentum in the decay, and  $S$  is the total spin of  $A$  plus  $B$ :

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \begin{pmatrix} J_B & J_C \\ \lambda_B & -\lambda_C \end{pmatrix} \begin{pmatrix} S \\ \lambda_B - \lambda_C \end{pmatrix} \times \begin{pmatrix} L & S \\ 0 & \lambda_B - \lambda_C \end{pmatrix} \begin{pmatrix} J_A \\ \lambda_B - \lambda_C \end{pmatrix}, \quad (2)$$

where the expressions in parentheses are the standard Wigner  $3j$  symbols. For strong decays, possible  $L$  values are constrained by the conservation of parity ( $P$ ):  $P_A = P_B P_C (-1)^L$ .

Denoting  $J/\psi$  as  $\psi$ , the matrix element for the  $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$  decay sequence is

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi} D_{\lambda_{\Lambda_b^0}, \lambda_{\Lambda^*} - \lambda_\psi}^{1/2}(0, \theta_{\Lambda_b^0}, 0)^* \times \mathcal{H}_{\lambda_p, 0}^{\Lambda_n^* \rightarrow K^p} D_{\lambda_{\Lambda^*}, \lambda_p}^{J_{\Lambda_n^*}}(\phi_K, \theta_{\Lambda^*}, 0)^* R_{\Lambda_n^*}(m_{Kp}) \times D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu, \theta_\psi, 0)^*, \quad (3)$$

where the  $x$  axis, in the coordinates describing the  $\Lambda_b^0$  decay, is chosen to fix  $\phi_{\Lambda^*} = 0$ . The sum over  $n$  is due to many different  $\Lambda_n^*$  resonances contributing to the amplitude. Since the  $J/\psi$  decay is electromagnetic, the values of  $\Delta\lambda_\mu \equiv \lambda_{\mu^+} - \lambda_{\mu^-}$  are restricted to  $\pm 1$ .

There are four (six) independent complex  $\mathcal{H}_{\lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi}$  couplings to fit for each  $\Lambda_n^*$  resonance for  $J_{\Lambda_n^*} = \frac{1}{2}$  ( $> \frac{1}{2}$ ). They can be reduced to only one (three) free  $B_{L,S}$  coupling to fit if only the lowest (the lowest two) values of  $L$  are considered. The mass  $m_{Kp}$ , together with all decay angles entering Eq. (3),  $\theta_{\Lambda_b^0}$ ,  $\theta_{\Lambda^*}$ ,  $\phi_K$ ,  $\theta_\psi$ , and  $\phi_\mu$  (denoted collectively as  $\Omega$ ), constitute the six independent dimensions of the  $\Lambda_b^0 \rightarrow J/\psi p K^-$  decay phase space.

Similarly, the matrix element for the  $P_c^+$  decay chain is given by

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{P_c} \equiv \sum_j \sum_{\lambda_{P_c}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_{P_c}, 0}^{\Lambda_b^0 \rightarrow P_{cj} K} D_{\lambda_{\Lambda_b^0}, \lambda_{P_c}}^{1/2}(\phi_{P_c}, \theta_{\Lambda_b^0}^0, 0)^* \times \mathcal{H}_{\lambda_\psi, \lambda_p}^{P_{cj} \rightarrow \psi p} D_{\lambda_{P_c}, \lambda_\psi - \lambda_p}^{J_{P_{cj}}}(\phi_\psi, \theta_{P_c}, 0)^* R_{P_{cj}}(m_{\psi p}) \times D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu^c, \theta_\psi^c, 0)^*, \quad (4)$$

where the angles and helicity states carry the superscript or subscript  $P_c$  to distinguish them from those defined for the  $\Lambda^*$  decay chain. The sum over  $j$  allows for the possibility of contributions from more than one  $P_c^+$  resonance. There are two (three) independent helicity couplings  $\mathcal{H}_{\lambda_\psi, \lambda_p}^{P_{cj} \rightarrow \psi p}$  for  $J_{P_{cj}} = \frac{1}{2}$  ( $> \frac{1}{2}$ ), and a ratio of the two  $\mathcal{H}_{\lambda_{P_c}, 0}^{\Lambda_b^0 \rightarrow P_{cj} K}$  couplings, to determine from the data.

The mass-dependent  $R_{\Lambda_n^*}(m_{Kp})$  and  $R_{P_{cj}}(m_{J/\psi p})$  terms are given by

$$R_X(m) = B'_{L_X}^X(p, p_0, d) \left( \frac{p}{M_{\Lambda_b^0}^X} \right)^{L_X^0} \times \text{BW}(m|M_{0X}, \Gamma_{0X}) B'_{L_X}(q, q_0, d) \left( \frac{q}{M_{0X}} \right)^{L_X}. \quad (5)$$

Here  $p$  is the  $X = \Lambda^*$  or  $P_c^+$  momentum in the  $\Lambda_b^0$  rest frame, and  $q$  is the momentum of either decay product of  $X$  in the  $X$  rest frame. The symbols  $p_0$  and  $q_0$  denote values of these quantities at the resonance peak ( $m = M_{0X}$ ). The orbital angular momentum between the decay products of  $\Lambda_b^0$  is denoted as  $L_{\Lambda_b^0}^X$ . Similarly,  $L_X$  is the orbital angular momentum between the decay products of  $X$ . The orbital angular momentum barrier factors,  $p^L B'_L(p, p_0, d)$ , involve the Blatt-Weisskopf functions [29], and account for the difficulty in creating larger orbital angular momentum  $L$ , which depends on the momentum of the decay products  $p$  and on the size of the decaying particle, given by the  $d$  constant. We set  $d = 3.0 \text{ GeV}^{-1} \sim 0.6 \text{ fm}$ . The relativistic Breit-Wigner amplitude is given by

$$\text{BW}(m|M_{0X}, \Gamma_{0X}) = \frac{1}{M_{0X}^2 - m^2 - iM_{0X}\Gamma(m)}, \quad (6)$$

where

$$\Gamma(m) = \Gamma_{0X} \left( \frac{q}{q_0} \right)^{2L_X+1} \frac{M_{0X}}{m} B'_{L_X}(q, q_0, d)^2 \quad (7)$$

is the mass-dependent width of the resonance. For the  $\Lambda(1405)$  resonance, which peaks below the  $K^- p$  threshold, we use a two-component Flatté-like parametrization [30]



(see the Supplemental Material [20]). The couplings for the allowed channels,  $\Sigma\pi$  and  $K\rho$ , are taken to be equal and to correspond to the nominal value of the width [12]. For all resonances we assume minimal values of  $L_{\Lambda_b^0}^X$  and of  $L_X$  in  $R_X(m)$ . For nonresonant (NR) terms we set  $\text{BW}(m) = 1$  and  $M_{\text{NR}}$  to the midrange mass.

Before the matrix elements for the two decay sequences can be added coherently, the proton and muon helicity states in the  $\Lambda^*$  decay chain must be expressed in the basis of helicities in the  $P_c^+$  decay chain,

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} + e^{i\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d_{\lambda_p^{P_c}, \lambda_p}^{1/2}(\theta_p) \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu}^{P_c} \right|^2, \quad (8)$$

where  $\theta_p$  is the polar angle in the  $p$  rest frame between the boost directions from the  $\Lambda^*$  and  $P_c^+$  rest frames, and  $\alpha_\mu$  is the azimuthal angle correcting for the difference between the muon helicity states in the two decay chains. Note that  $m_{\psi p}$ ,  $\theta_{\Lambda_b^0}^{P_c}$ ,  $\phi_{P_c}$ ,  $\theta_{P_c}$ ,  $\phi_\psi$ ,  $\theta_\psi^{P_c}$ ,  $\phi_\mu^{P_c}$ ,  $\theta_p$ , and  $\alpha_\mu$  can all be derived from the values of  $m_{K\rho}$  and  $\Omega$ , and thus do not constitute independent dimensions in the  $\Lambda_b^0$  decay phase space. (A detailed prescription for calculation of all the angles entering the matrix element is given in the Supplemental Material [20].)

Strong interactions, which dominate  $\Lambda_b^0$  production at the LHC, conserve parity and cannot produce longitudinal  $\Lambda_b^0$  polarization [31]. Therefore,  $\lambda_{\Lambda_b^0} = +1/2$  and  $-1/2$  values are equally likely, which is reflected in Eq. (8). If we allow the  $\Lambda_b^0$  polarization to vary, the data are consistent with a polarization of zero. Interferences between various  $\Lambda_n^*$  and  $P_{cj}^+$  resonances vanish in the integrated rates unless the resonances belong to the same decay chain and have the same quantum numbers.

The matrix element given by Eq. (8) is a six-dimensional function of  $m_{K\rho}$  and  $\Omega$  and depends on the fit parameters,  $\vec{\omega}$ , which represent independent helicity or  $LS$  couplings, and masses and widths of resonances (or Flatté parameters),  $\mathcal{M} = \mathcal{M}(m_{K\rho}, \Omega | \vec{\omega})$ . After accounting for the selection efficiency to obtain the signal probability density function (PDF), an unbinned maximum likelihood fit is used to determine the amplitudes. Since the efficiency does not depend on  $\vec{\omega}$ , it is needed only in the normalization integral, which is carried out numerically by summing  $|\mathcal{M}(m_{K\rho}, \Omega | \vec{\omega})|^2$  over the simulated events generated uniformly in phase space and passed through the selection. (More details are given in the Supplemental Material [20].)

We use two fit algorithms, which were independently coded and which differ in the approach used for background subtraction. In the first approach, which we refer to

as cFit, the signal region is defined as  $\pm 2\sigma$  around the  $\Lambda_b^0$  mass peak. The total PDF used in the fit to the candidates in the signal region,  $\mathcal{P}(m_{K\rho}, \Omega | \vec{\omega})$ , includes a background component with normalization fixed to be 5.4% of the total. The background PDF is found to factorize into five two-dimensional functions of  $m_{K\rho}$  and of each independent angle, which are estimated using sidebands extending from  $5.0\sigma$  to  $13.5\sigma$  on both sides of the peak.

In the complementary approach, called sFit, no explicit background parametrization is needed. The PDF consists of only the signal component, with the background subtracted using the sPlot technique [32] applied to the log-likelihood sum. All candidates shown in Fig. 4 are included in the sum with weights,  $W_i$ , dependent on  $m_{J/\psi K\rho}$ . The weights are set according to the signal and the background probabilities determined by the fits to the  $m_{J/\psi pK}$  distributions, similar to the fit displayed in Fig. 4, but performed in 32 different bins of the two-dimensional plane of  $\cos\theta_{\Lambda_b^0}$  and  $\cos\theta_{J/\psi}$  to account for correlations with the mass shapes of the signal and background components. This quasi-log-likelihood sum is scaled by a constant factor,  $s_W \equiv \sum_i W_i / \sum_i W_i^2$ , to account for the effect of the background subtraction on the statistical uncertainty. (More details on the cFit and sFit procedures are given in the Supplemental Material [20].)

In each approach, we minimize  $-2 \ln \mathcal{L}(\vec{\omega}) = -2s_W \sum_i W_i \ln \mathcal{P}(m_{K\rho}, \Omega_i | \vec{\omega})$ , which gives the estimated values of the fit parameters,  $\vec{\omega}_{\text{min}}$ , together with their covariance matrix ( $W_i = 1$  in cFit). The difference of  $-2 \ln \mathcal{L}(\vec{\omega}_{\text{min}})$  between different amplitude models,  $\Delta(-2 \ln \mathcal{L})$ , allows their discrimination. For two models representing separate hypotheses, e.g., when discriminating between different  $J^P$  values assigned to a  $P_c^+$  state, the assumption of a  $\chi^2$  distribution with 1 degree of freedom for  $\Delta(-2 \ln \mathcal{L})$  under the disfavored  $J^P$  hypothesis allows the calculation of a lower limit on the significance of its rejection, i.e., the  $p$  value [33]. Therefore, it is convenient to express  $\Delta(-2 \ln \mathcal{L})$  values as  $n_\sigma^2$ , where  $n_\sigma$  corresponds to the number of standard deviations in the normal distribution with the same  $p$  value. For nested hypotheses, e.g., when discriminating between models without and with  $P_c^+$  states,  $n_\sigma$  overestimates the  $p$  value by a modest amount. Simulations are used to obtain better estimates of the significance of the  $P_c^+$  states.

Since the isospin of both the  $\Lambda_b^0$  and the  $J/\psi$  particles are zero, we expect that the dominant contributions in the  $K^- p$  system are  $\Lambda^*$  states, which would be produced via a  $\Delta I = 0$  process. It is also possible that  $\Sigma^*$  resonances contribute, but these would have  $\Delta I = 1$ . By analogy with kaon decays the  $\Delta I = 0$  process should be dominant [34]. The list of  $\Lambda^*$  states considered is shown in Table I.

Our strategy is to first try to fit the data with a model that can describe the mass and angular distributions including only  $\Lambda^*$  resonances, allowing all possible known states and decay amplitudes. We call this the “extended” model. It has

TABLE I. The  $\Lambda^*$  resonances used in the different fits. Parameters are taken from the PDG [12]. We take  $5/2^-$  for the  $J^P$  of the  $\Lambda(2585)$ . The number of  $LS$  couplings is also listed for both the reduced and extended models. To fix overall phase and magnitude conventions, which otherwise are arbitrary, we set  $B_{0\frac{1}{2}} = (1, 0)$  for  $\Lambda(1520)$ . A zero entry means the state is excluded from the fit.

State	$J^P$	$M_0$ (MeV)	$\Gamma_0$ (MeV)	Number Reduced	Number Extended
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	$50.5 \pm 2.0$	3	4
$\Lambda(1520)$	$3/2^-$	$1519.5 \pm 1.0$	$15.6 \pm 1.0$	5	6
$\Lambda(1600)$	$1/2^+$	1600	150	3	4
$\Lambda(1670)$	$1/2^-$	1670	35	3	4
$\Lambda(1690)$	$3/2^-$	1690	60	5	6
$\Lambda(1800)$	$1/2^-$	1800	300	4	4
$\Lambda(1810)$	$1/2^+$	1810	150	3	4
$\Lambda(1820)$	$5/2^+$	1820	80	1	6
$\Lambda(1830)$	$5/2^-$	1830	95	1	6
$\Lambda(1890)$	$3/2^+$	1890	100	3	6
$\Lambda(2100)$	$7/2^-$	2100	200	1	6
$\Lambda(2110)$	$5/2^+$	2110	200	1	6
$\Lambda(2350)$	$9/2^+$	2350	150	0	6
$\Lambda(2585)$	?	$\approx 2585$	200	0	6

146 free parameters from the helicity couplings alone. The masses and widths of the  $\Lambda^*$  states are fixed to their PDG values, since allowing them to float prevents the fit from converging. Variations in these parameters are considered in the systematic uncertainties.

The cFit results without any  $P_c^+$  component are shown in Fig. 6. While the  $m_{Kp}$  distribution is reasonably well fitted, the peaking structure in  $m_{J/\psi p}$  is not reproduced. The same result is found using sFit. The speculative addition of  $\Sigma^*$  resonances to the states decaying to  $K^- p$  does not change this conclusion.

We will demonstrate that introducing two  $P_c^+ \rightarrow J/\psi p$  resonances leads to a satisfactory description of the data. When determining parameters of the  $P_c^+$  states, we use a more restrictive model of the  $K^- p$  states (hereafter referred

to as the “reduced” model) that includes only the  $\Lambda^*$  resonances that are well motivated, and has fewer than half the number of free parameters. As the minimal  $L_{\Lambda_b^0}^{\Lambda^*}$  for the spin  $9/2$   $\Lambda(2350)$  equals  $J_{\Lambda^*} - J_{\Lambda_b^0} - J_{J/\psi} = 3$ , it is extremely unlikely that this state can be produced so close to the phase space limit. In fact  $L = 3$  is the highest orbital angular momentum observed, with a very small rate, in decays of  $B$  mesons [35] with much larger phase space available ( $Q = 2366$  MeV, while here  $Q = 173$  MeV), and without additional suppression from the spin counting factors present in  $\Lambda(2350)$  production (all three  $\vec{J}_{\Lambda^*}$ ,  $\vec{J}_{\Lambda_b^0}$  and  $\vec{J}_{J/\psi}$  vectors have to line up in the same direction to produce the minimal  $L_{\Lambda_b^0}^{\Lambda^*}$  value). Therefore, we eliminate it

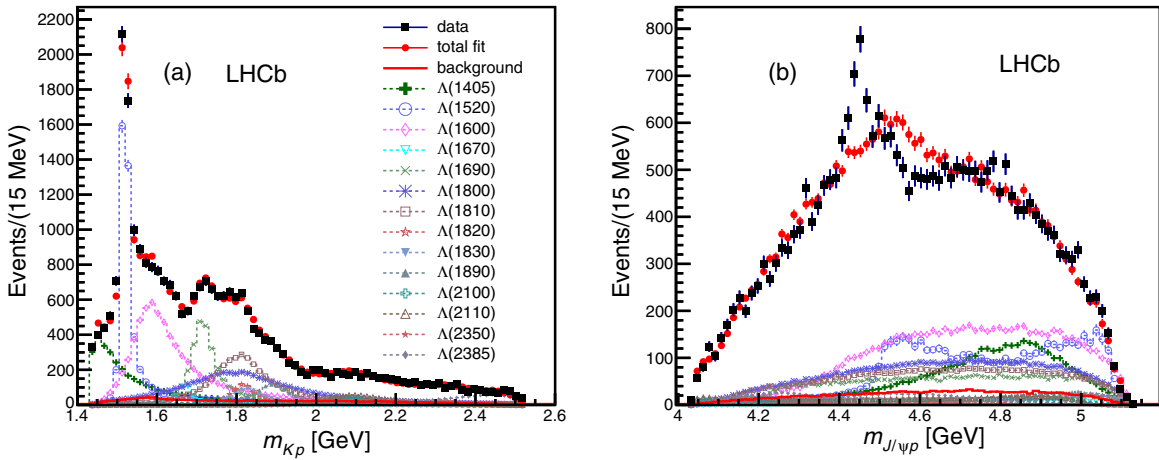


FIG. 6 (color online). Results for (a)  $m_{Kp}$  and (b)  $m_{J/\psi p}$  for the extended  $\Lambda^*$  model fit without  $P_c^+$  states. The data are shown as (black) squares with error bars, while the (red) circles show the results of the fit. The error bars on the points showing the fit results are due to simulation statistics.

from the reduced  $\Lambda^*$  model. We also eliminate the  $\Lambda(2585)$  state, which peaks beyond the kinematic limit and has unknown spin. The other resonances are kept but high  $L_{\Lambda_b^0}^{\Lambda^*}$  amplitudes are removed; only the lowest values are kept for the high mass resonances, with a smaller reduction for the lighter ones. The number of  $LS$  amplitudes used for each resonance is listed in Table I. With this model, we reduce the number of parameters needed to describe the  $\Lambda^*$  decays from 146 to 64. For the different combinations of  $P_c^+$  resonances that we try, there are up to 20 additional free parameters. Using the extended model including one resonant  $P_c^+$  improves the fit quality, but it is still unacceptable (see Supplemental Material [20]). We find acceptable fits with two  $P_c^+$  states. We use the reduced  $\Lambda^*$  model for the central values of our results. The differences in fitted quantities with the extended model are included in the systematic uncertainties.

The best fit combination finds two  $P_c^+$  states with  $J^P$  values of  $3/2^-$  and  $5/2^+$ , for the lower and higher mass

states, respectively. The  $-2 \ln \mathcal{L}$  values differ by only 1 unit between the best fit and the parity reversed combination ( $3/2^+, 5/2^-$ ). Other combinations are less likely, although the  $(5/2^+, 3/2^-)$  pair changes  $-2 \ln \mathcal{L}$  by only 2.3<sup>2</sup> units and therefore cannot be ruled out. All combinations  $1/2^\pm$  through  $7/2^\pm$  were tested, and all others are disfavored by changes of more than 5<sup>2</sup> in the  $-2 \ln \mathcal{L}$  values. The cFit results for the  $(3/2^-, 5/2^+)$  fit are shown in Fig. 3. Both distributions of  $m_{Kp}$  and  $m_{J/\psi p}$  are reproduced. The lower mass  $3/2^-$  state has mass  $4380 \pm 8$  MeV and width  $205 \pm 18$  MeV, while the  $5/2^+$  state has a mass of  $4449.8 \pm 1.7$  MeV and width  $39 \pm 5$  MeV. These errors are statistical only; systematic uncertainties are discussed later. The mass resolution is approximately 2.5 MeV and does not affect the width determinations. The sFit approach gives comparable results. The angular distributions are reasonably well reproduced, as shown in Fig. 7, and the comparison with the data in  $m_{Kp}$  intervals is also satisfactory as can be seen in Fig. 8. Interference effects between

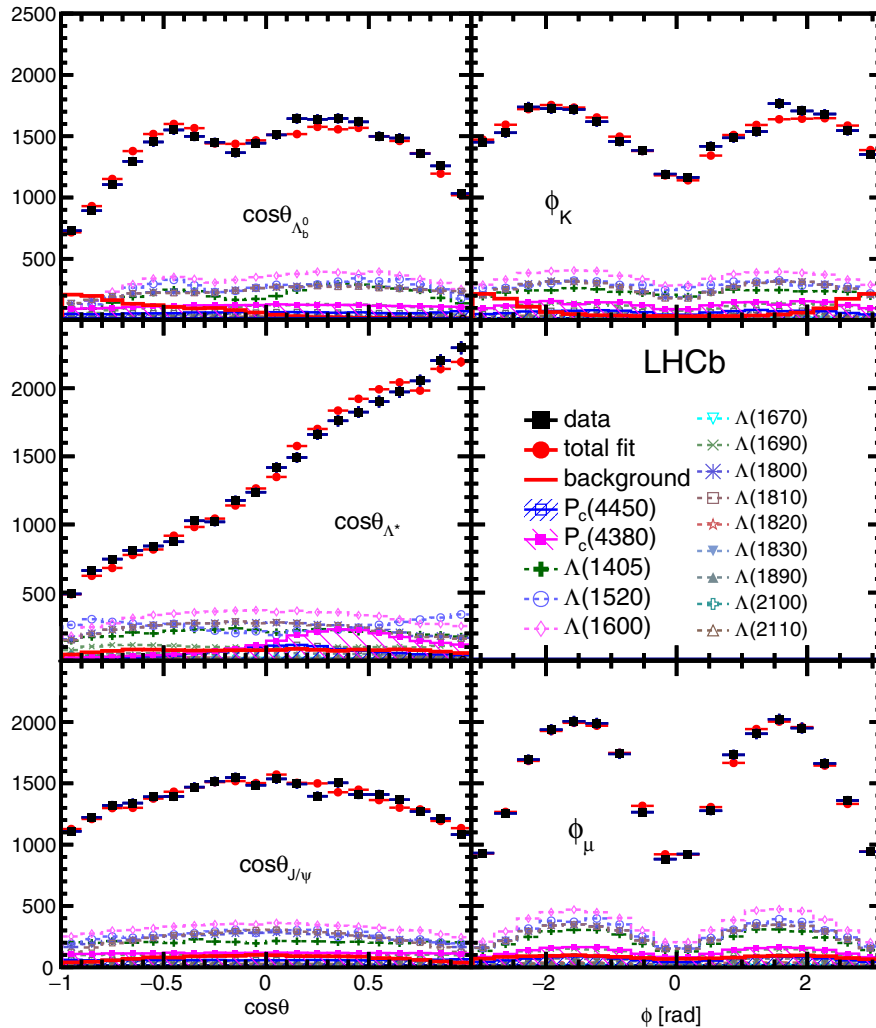


FIG. 7 (color online). Various decay angular distributions for the fit with two  $P_c^+$  states. The data are shown as (black) squares, while the (red) circles show the results of the fit. Each fit component is also shown. The angles are defined in the text.



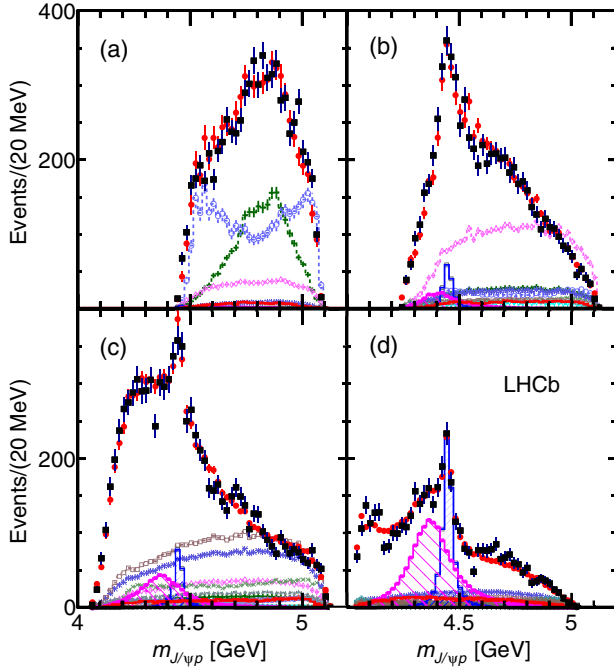


FIG. 8 (color online).  $m_{J/\psi p}$  in various intervals of  $m_{Kp}$  for the fit with two  $P_c^+$  states: (a)  $m_{Kp} < 1.55$  GeV, (b)  $1.55 < m_{Kp} < 1.70$  GeV, (c)  $1.70 < m_{Kp} < 2.00$  GeV, and (d)  $m_{Kp} > 2.00$  GeV. The data are shown as (black) squares with error bars, while the (red) circles show the results of the fit. The blue and purple histograms show the two  $P_c^+$  states. See Fig. 7 for the legend.

the two  $P_c^+$  states are particularly evident in Fig. 8(d), where there is a large destructive contribution (not explicitly shown in the figure) to the total rate. (A fit fraction comparison between cFit and sFit is given in the Supplemental Material [20].) The addition of further  $P_c^+$  states does not significantly improve the fit.

Adding a single  $5/2^+ P_c^+$  state to the fit with only  $\Lambda^*$  states reduces  $-2 \ln \mathcal{L}$  by  $14.7^2$  using the extended model and adding a second lower mass  $3/2^- P_c^+$  state results in a further reduction of  $11.6^2$ . The combined reduction of  $-2 \ln \mathcal{L}$  by the two states taken together is  $18.7^2$ . Since taking  $\sqrt{\Delta 2 \ln \mathcal{L}}$  overestimates significances, we perform simulations to obtain more accurate evaluations. We generate pseudoexperiments using the null hypotheses having amplitude parameters determined by the fits to the data with no or one  $P_c^+$  state. We fit each pseudoexperiment with the null hypothesis and with  $P_c^+$  states added to the model. The  $-2 \ln \mathcal{L}$  distributions obtained from many pseudoexperiments are consistent with  $\chi^2$  distributions with the number of degrees of freedom approximately equal to twice the number of extra parameters in the fit. Comparing these distributions with the  $\Delta 2 \ln \mathcal{L}$  values from the fits to the data,  $p$  values can be calculated. These studies show reduction of the significances relative to  $\sqrt{\Delta 2 \ln \mathcal{L}}$  by about 20%, giving overall significances of  $9\sigma$  and  $12\sigma$ ,

for the lower and higher mass  $P_c^+$  states, respectively. The combined significance of two  $P_c^+$  states is  $15\sigma$ . Use of the extended model to evaluate the significance includes the effect of systematic uncertainties due to the possible presence of additional  $\Lambda^*$  states or higher  $L$  amplitudes.

Systematic uncertainties are evaluated for the masses, widths, and fit fractions of the  $P_c^+$  states, and for the fit fractions of the two lightest and most significant  $\Lambda^*$  states. Additional sources of modeling uncertainty that we have not considered may affect the fit fractions of the heavier  $\Lambda^*$  states. The sources of systematic uncertainties are listed in Table II. They include differences between the results of the extended versus reduced model, varying the  $\Lambda^*$  masses and widths, uncertainties in the identification requirements for the proton and restricting its momentum, inclusion of a nonresonant amplitude in the fit, use of separate higher and lower  $\Lambda_b^0$  mass sidebands, alternate  $J^P$  fits, varying the Blatt-Weisskopf barrier factor  $d$  between 1.5 and  $4.5 \text{ GeV}^{-1}$ , changing the angular momentum  $L$  used in Eq. (5) by one or two units, and accounting for potential mismodeling of the efficiencies. For the  $\Lambda(1405)$  fit fraction we also added an uncertainty for the Flatté couplings, determined by both halving and doubling their ratio, and taking the maximum deviation as the uncertainty.

The stability of the results is cross-checked by comparing the data recorded in 2011 (2012), with the LHCb dipole magnet polarity in up (down) configurations,  $\Lambda_b^0(\bar{\Lambda}_b^0)$  decays, and  $\Lambda_b^0$  produced with low (high) values of  $p_T$ . Extended model fits without including  $P_c^+$  states were tried with the addition of two high mass  $\Lambda^*$  resonances of freely varied mass and width, or four nonresonant components up to spin  $3/2$ ; these do not explain the data. The fitters were tested on simulated pseudoexperiments and no biases were found. In addition, selection requirements are varied, and the vetoes of  $\bar{B}_s^0$  and  $\bar{B}^0$  are removed and explicit models of those backgrounds added to the fit; all give consistent results.

Further evidence for the resonant character of the higher mass, narrower  $P_c^+$  state is obtained by viewing the evolution of the complex amplitude in the Argand diagram [12]. In the amplitude fits discussed above, the  $P_c(4450)^+$  is represented by a Breit-Wigner amplitude, where the magnitude and phase vary with  $m_{J/\psi p}$  according to an approximately circular trajectory in the  $(\text{Re}A^{P_c}, \text{Im}A^{P_c})$  plane, where  $A^{P_c}$  is the  $m_{J/\psi p}$  dependent part of the  $P_c(4450)^+$  amplitude. We perform an additional fit to the data using the reduced  $\Lambda^*$  model, in which we represent the  $P_c(4450)^+$  amplitude as the combination of independent complex amplitudes at six equidistant points in the range  $\pm \Gamma_0 = 39 \text{ MeV}$  around  $M_0 = 4449.8 \text{ MeV}$  as determined in the default fit. Real and imaginary parts of the amplitude are interpolated in the mass interval between the fitted points. The resulting Argand diagram, shown in Fig. 9(a), is consistent with a rapid counterclockwise change of the  $P_c(4450)^+$  phase when its magnitude reaches

TABLE II. Summary of systematic uncertainties on  $P_c^+$  masses, widths, and fit fractions, and  $\Lambda^*$  fit fractions. A fit fraction is the ratio of the phase space integrals of the matrix element squared for a single resonance and for the total amplitude. The terms “low” and “high” correspond to the lower and higher mass  $P_c^+$  states. The sFit–cFit difference is listed as a cross-check and not included as an uncertainty.

Source	$M_0$ (MeV)		$\Gamma_0$ (MeV)		Fit Fractions (%)			
	Low	High	Low	High	Low	High	$\Lambda(1405)$	$\Lambda(1520)$
Extended versus reduced	21	0.2	54	10	3.14	0.32	1.37	0.15
$\Lambda^*$ masses and widths	7	0.7	20	4	0.58	0.37	2.49	2.45
Proton ID	2	0.3	1	2	0.27	0.14	0.20	0.05
$10 < p_p < 100$ GeV	0	1.2	1	1	0.09	0.03	0.31	0.01
Nonresonant	3	0.3	34	2	2.35	0.13	3.28	0.39
Separate sidebands	0	0	5	0	0.24	0.14	0.02	0.03
$J^P(3/2^+, 5/2^-)$ or $(5/2^+, 3/2^-)$	10	1.2	34	10	0.76	0.44		
$d = 1.5\text{--}4.5\text{GeV}^{-1}$	9	0.6	19	3	0.29	0.42	0.36	1.91
$L_{\Lambda_b^0}^{P_c} \Lambda_b^0 \rightarrow P_c^+ (\text{low or high}) K^-$	6	0.7	4	8	0.37	0.16		
$L_{P_c} P_c^+ (\text{low or high}) \rightarrow J/\psi p$	4	0.4	31	7	0.63	0.37		
$L_{\Lambda_b^0}^{\Lambda^*} \Lambda_b^0 \rightarrow J/\psi \Lambda^*$	11	0.3	20	2	0.81	0.53	3.34	2.31
Efficiencies	1	0.4	4	0	0.13	0.02	0.26	0.23
Change $\Lambda(1405)$ coupling	0	0	0	0	0	0	1.90	0
Overall	29	2.5	86	19	4.21	1.05	5.82	3.89
sFit/cFit cross-check	5	1.0	11	3	0.46	0.01	0.45	0.13

the maximum, a behavior characteristic of a resonance. A similar study for the wider state is shown in Fig. 9(b); although the fit does show a large phase change, the amplitude values are sensitive to the details of the  $\Lambda^*$  model and so this latter study is not conclusive.

Different binding mechanisms of pentaquark states are possible. Tight binding was envisioned originally [3,4,36]. A possible explanation is heavy-light diquarks [37]. Examples of other mechanisms include a

diquark-diquark-antiquark model [38,39], a diquark-triquark model [40], and a coupled channel model [41]. Weakly bound “molecules” of a baryon plus a meson have been also discussed [42].

Models involving thresholds or “cusps” have been invoked to explain some exotic meson candidates via nonresonant scattering mechanisms [43–45]. There are certain obvious difficulties with the use of this approach to explain our results. The closest threshold to the high

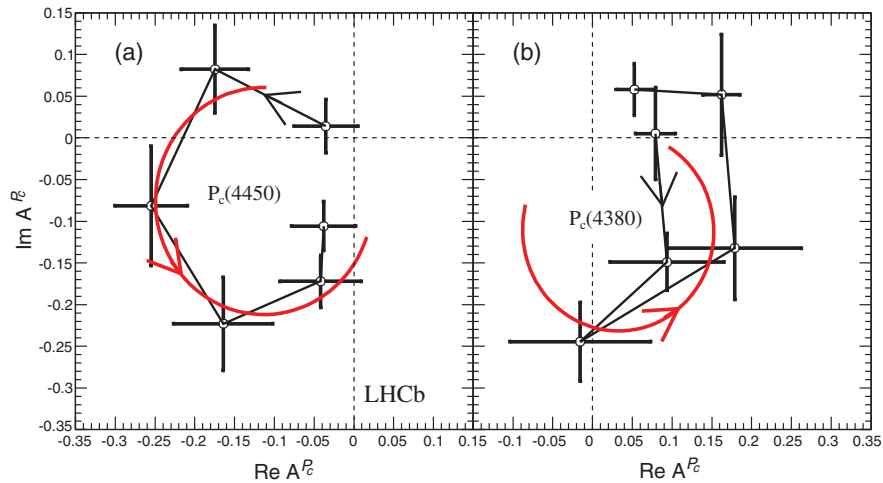


FIG. 9 (color online). Fitted values of the real and imaginary parts of the amplitudes for the baseline  $(3/2^-, 5/2^+)$  fit for (a) the  $P_c(4450)^+$  state and (b) the  $P_c(4380)^+$  state, each divided into six  $m_{J/\psi p}$  bins of equal width between  $-\Gamma_0$  and  $+\Gamma_0$  shown in the Argand diagrams as connected points with error bars ( $m_{J/\psi p}$  increases counterclockwise). The solid (red) curves are the predictions from the Breit-Wigner formula for the same mass ranges with  $M_0$  ( $\Gamma_0$ ) of 4450 (39) MeV and 4380 (205) MeV, respectively, with the phases and magnitudes at the resonance masses set to the average values between the two points around  $M_0$ . The phase convention sets  $B_{0, \frac{1}{2}} = (1, 0)$  for  $\Lambda(1520)$ . Systematic uncertainties are not included.

mass state is at  $4457.1 \pm 0.3$  MeV resulting from a  $\Lambda_c(2595)^+\bar{D}^0$  combination, which is somewhat higher than the peak mass value and would produce a structure with quantum numbers  $J^P = 1/2^+$  which are disfavored by our data. There is no threshold close to the lower mass state.

In conclusion, we have presented a full amplitude fit to the  $\Lambda_b^0 \rightarrow J/\psi K^- p$  decay. We observe significant  $\Lambda^*$  production recoiling against the  $J/\psi$  with the lowest mass contributions, the  $\Lambda(1405)$  and  $\Lambda(1520)$  states having fit fractions of  $(15 \pm 1 \pm 6)\%$  and  $(19 \pm 1 \pm 4)\%$ , respectively. The data cannot be satisfactorily described without including two Breit-Wigner shaped resonances in the  $J/\psi p$  invariant mass distribution. The significances of the lower mass and higher mass states are 9 and 12 standard deviations, respectively. These structures cannot be accounted for by reflections from  $J/\psi \Lambda^*$  resonances or other known sources. Interpreted as resonant states they must have minimal quark content of  $c\bar{c}uud$ , and would therefore be called charmonium-pentaquark states. The lighter state  $P_c(4380)^+$  has a mass of  $4380 \pm 8 \pm 29$  MeV and a width of  $205 \pm 18 \pm 86$  MeV, while the heavier state  $P_c(4450)^+$  has a mass of  $4449.8 \pm 1.7 \pm 2.5$  MeV and a width of  $39 \pm 5 \pm 19$  MeV. A model-independent representation of the  $P_c(4450)^+$  contribution in the fit shows a phase change in amplitude consistent with that of a resonance. The parities of the two states are opposite with the preferred spins being  $3/2$  for one state and  $5/2$  for the other. The higher mass state has a fit fraction of  $(4.1 \pm 0.5 \pm 1.1)\%$ , and the lower mass state of  $(8.4 \pm 0.7 \pm 4.2)\%$ , of the total  $\Lambda_b^0 \rightarrow J/\psi K^- p$  sample.

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